# Circle's folding up into a Cone using Cylindrical Coordinate System

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October 13, 2023

## 1 Circle's folding up into a Cone using Cylindrical Coordinate System

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**Task**

**"My project is to take the circle's folding up into a cone using the coordinate system in: https://zenodo.org/record/7710313 from the difference between the circumferences of two circles, and have a mathematica notebook that animates the transformation using the coordinate systems specifically."**

Cone nets are possible for the vertex angle of $\pi/2$,, $3\pi/2$,$2\pi$ and so on. The transformation from a point $(x, y)$ in the grid plane to the point $(x', y', z')$ on the cone is given by

$$x = s\cos(\theta) \tag{1}$$

$$y = s\sin(\theta) \tag{2}$$

$$z = h - hs/r_1. \tag{3}$$

Let

$$\begin{aligned} z &= h - \frac{h}{r_1}s \\ z - h &= -s\frac{h}{r_1}s \\ s &= (1 - \frac{z}{h})r_1 \end{aligned}$$

A maximum $h = 1$, so we can take

$$s = (1 - z)r_1 \tag{4}$$

Put the value of (4) into (3) we get

$$Z = h - \frac{h}{r_1}(1-z)r_1 z = h - h + hz = hz \tag{5}$$

So $Z$ will be scalled with $zh$ Thus our new system is

$$x' = (1-z)r_1\cos(\theta) \tag{6}$$
$$y' = (1-z)r_1\sin(\theta) \tag{7}$$
$$z' = zh. \tag{8}$$

**The difference of circumference of two circles**

The difference of circumference of the two circles is

$$r\theta = 2\pi r - 2\pi r_1 \tag{9}$$

Since we have taken $\theta = \pi + h\pi = \pi(1+h)$ substitute $\theta$ in (9)

$$\frac{r\pi(1+h)}{2} = \pi r - \pi r_1 \tag{10}$$

Using pythagorean theorem for cone with radius $r_1$ and slant hight $r$ i.e

$$r = \sqrt{h^2 + r_1^2} \tag{11}$$

Replacing $r$, we get,

$$\begin{aligned}
\sqrt{h^2+r_1^2}\frac{\pi(1+h)}{2} &= \pi(\sqrt{h^2+r_1^2} - r_1) \\
\frac{\pi(1+h)}{2} &= \pi\left(1 - \frac{r_1}{\sqrt{h^2+r_1^2}}\right) \\
\frac{\pi(1+h)}{2} + \frac{\pi r_1}{\sqrt{h^2+r_1^2}} &= \pi \\
\frac{\pi(1+h)}{2}\sqrt{h^2+r_1^2} + 2\pi r_1 &= \pi r \\
\frac{\pi(1+h)}{2}\sqrt{h^2+r_1^2} &= \pi r - 2\pi r_1
\end{aligned}$$

Multiplying both sides by $r_1$, we get,

$$\frac{\pi r_1(1+h)}{2}\sqrt{h^2+r_1^2} = \pi r r_1 - 2\pi r_1^2 \tag{12}$$

When $h \to 0$ in right side of the last equation we have only the circle's area i.e. $\pi r^2$ so we can use this relation to draw a cone of angle $\pi/2, 2\pi, 3\pi/2, 2\pi$ and so

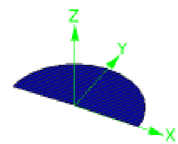
on
We plot parametric equations (6),(7) and (8) with relation (12) by using ParametricPlot3D in Mathematica.
So when h=0 we only have circle in 2D. We will use (6), (7) and (8) as a parametric equations to solve for cone. In which s was a radius of a projection of Surface. For simplicity of calculations. we transformed this in terms of Z and r1. And r1 will be Calculated from iterative process from (12). Where r1 takes inputs from r and $\theta$ i.e initial parameters of circle. Whenever we increase height from 0 to some number in our case 0 to 1 cone will be generated.
For each fixed h we have some condition $\theta = \pi + h\pi$ and $r$ is calculated from (9) a difference of circumference of two circles as per your requirement.
Note: Radius is the function of height.

Mathematica Code

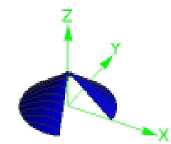
```
ClearAll[AnimatedCone, Radius]
Radius[h_] = r /. First@
    Solve[{Pi r Sqrt[r^2 + h^2] (1 + h )/2 == Pi /2, h > 0}, r, Reals];
Radius[0] = 1;
AnimatedCone[h_] := ParametricPlot3D[{Cos[θ] (1 - z) Radius[h], Sin[θ] (1 - z) Radius[h], h z},
   {θ, 0, Pi + h Pi}, {z, 0, 1},
   PlotStyle → Blue,
   MeshFunctions → {#2&},
   Boxed → False,
   PlotRange → 1.5 {{-2, 2}, {-2, 2}, {0, 1}},
   BoundaryStyle → GrayLevel[.1],
   PerformanceGoal → "Quality",
   Axes → False];
axes = Graphics3D[{Green, Arrowheads[Medium],
    MapThread[{Arrow[{{0, 0, 0}, 1.4 #2}], Text[#, 1.5 #2]} &,
     {{"X", "Y", "Z"}, IdentityMatrix[3]}]}];
Manipulate[Show[AnimatedCone @ h, axes], {{h, 0, "height"}, 0, 1}]
```

Adapted AI transform:

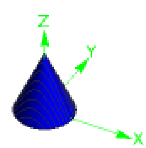
```
ClearAll[AnimatedCone, Radius, θ, x, β]
Radius[x_, h_] = r /. First@Solve[{2 π r - 2 π Sqrt[r^2 - h^2] == x - h, h > 0}, r, Reals];
Radius[x_, 0] = x;
θ[x_, h_] =
  (2 (π r - π x)) / r /. First@Solve[{2 π r - 2 π Sqrt[r^2 - h^2] == x - h, h > 0}, r, Reals];
β[h_] = ArcSin[Sqrt[(4 π - θ[x, h]) θ[x, h]] / (2 π)];
AnimatedCone[x_, h_] :=
  ParametricPlot3D[{Cos[θ[x, h]] (1 - z) Radius[x, h], Sin[θ[x, h]] (1 - z) Radius[x, h],
    h z}, {θ[x, h], 0, π + h π}, {z, 0, 1}, PlotStyle → Blue, MeshFunctions → {#2 &},
   Boxed → False, PlotRange → 1.5 {{-2, 2}, {-2, 2}, {0, 1}}, BoundaryStyle → GrayLevel[.1],
   PerformanceGoal → "Quality", Axes → False];
axes =
  Graphics3D[{Green, Arrowheads[Medium],
    MapThread[{Arrow[{{0, 0, 0}, 1.4 #2}], Text[#, 1.5 #2]} &,
     {{"X", "Y", "Z"}, IdentityMatrix[3]}]}];
Manipulate[Show[AnimatedCone@x, axes], {{x, 2, "Base"}, 0.2, 3}, {{h, 0, "height"}, 0, 3},
 {{α, 0, "angle"}, 0, 2 π}, ControlType → Slider]
```




(a) A figure1




(b) A figure2




(c) A figure3

Figure 1: A captured figures of animation